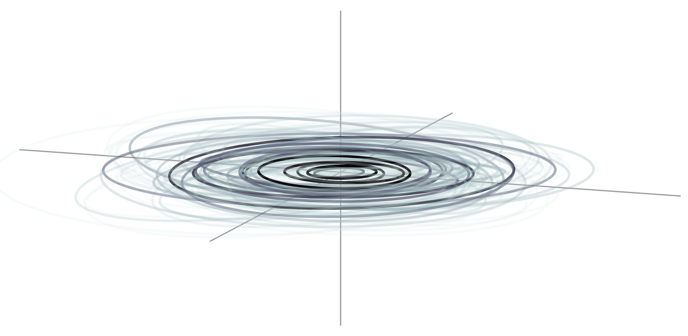
Chaos in Terrestrial Planet Formation



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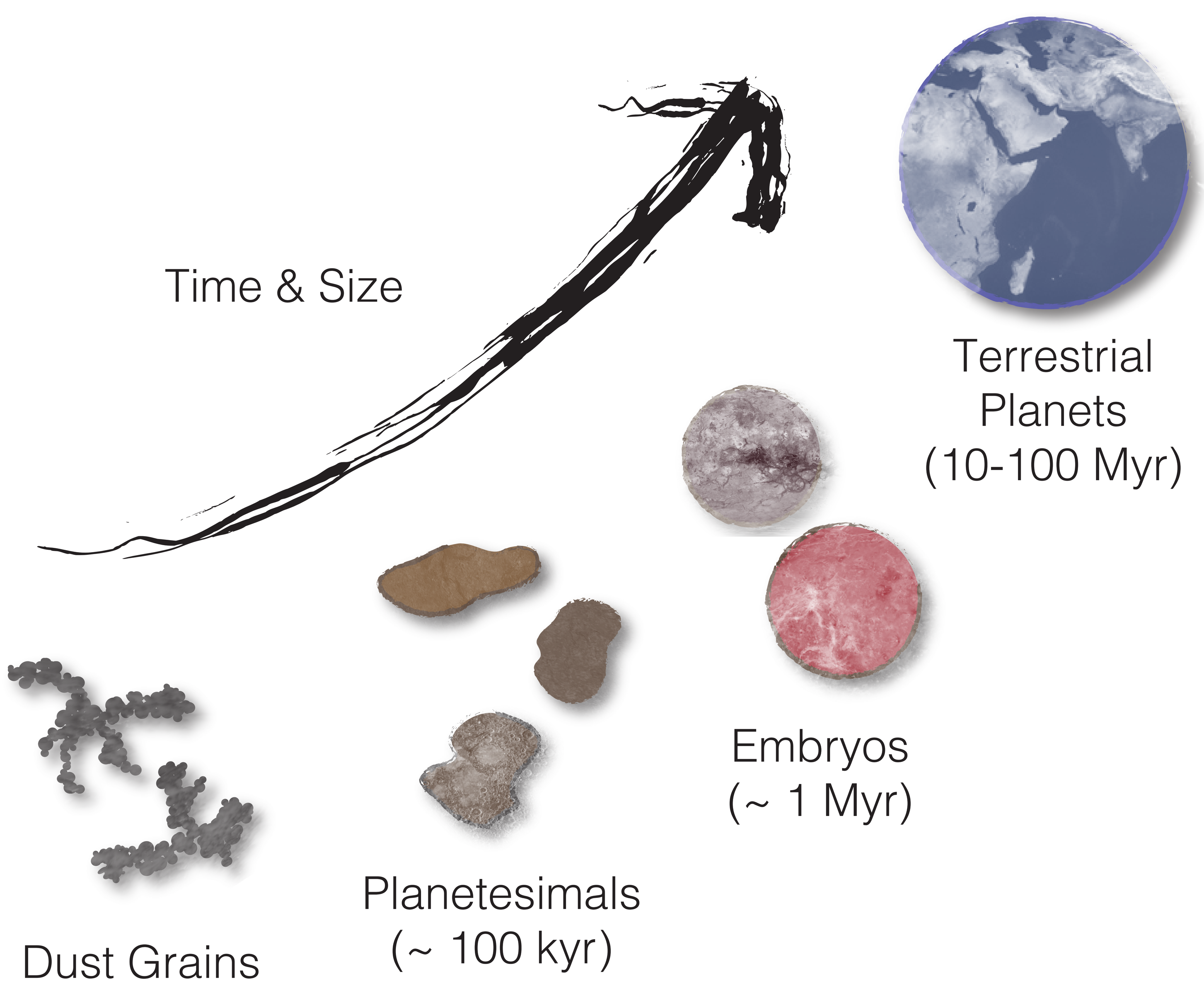
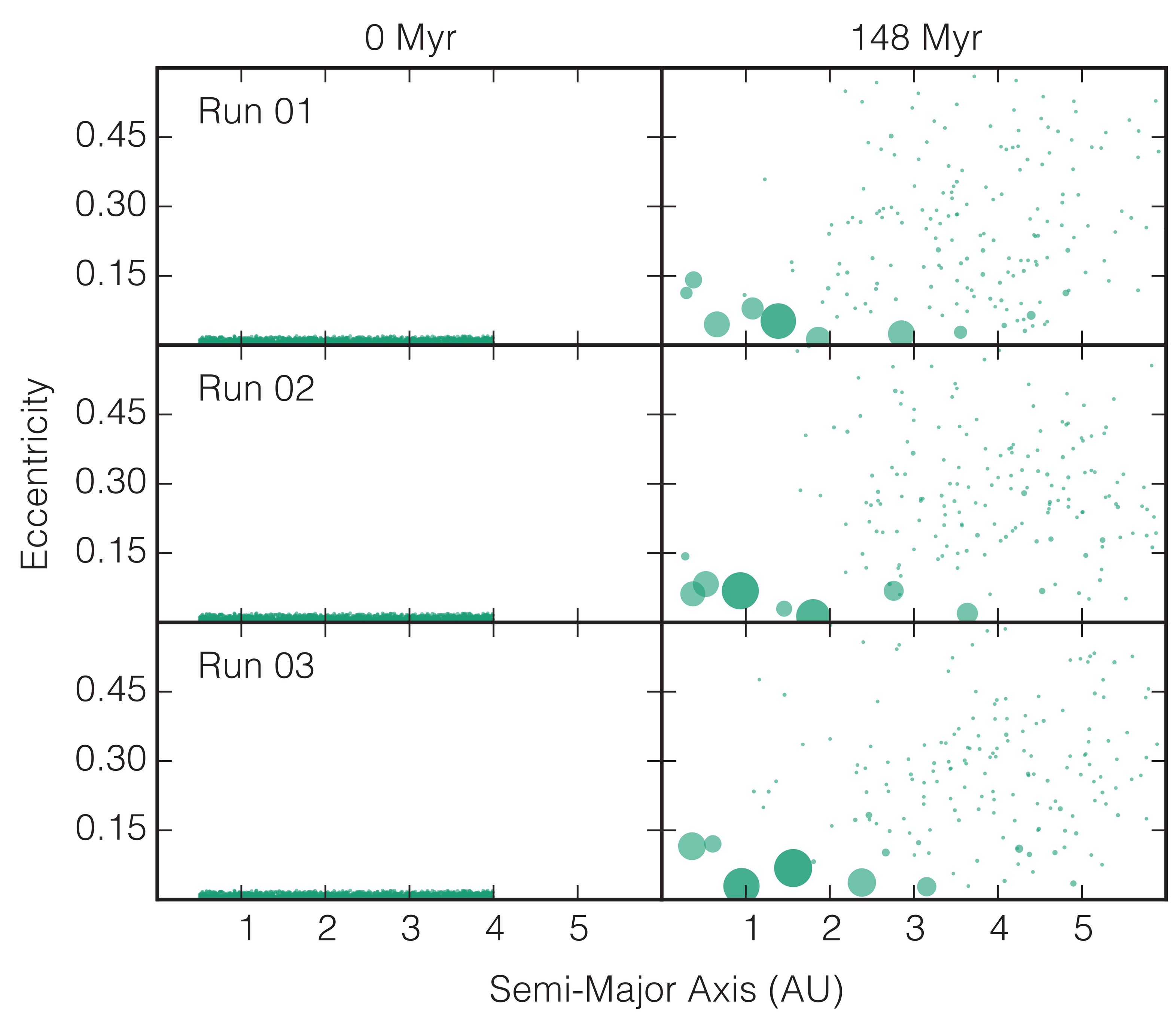
Planet Formation 101

Planets form in gaseous disks surrounding young stars. Embedded in these evaporating disks are micron sized dust grains. They assemble into aggregates, grow to km-sized planetesimals, form moon-sized planetary embryos, and finally terrestrial planets (Raymond+ 2013). This is sketched on the right.

To date, no codes tackle the entire formation process, and research relies on a piecewise approach. Fluid dynamics codes investigate the gaseous disk, Monte-Carlo coagulation models track growth of aggregates, and N-Body codes address the evolution from planetesimals to planets. We focus on the latter.

N-Body codes require (i) efficient processing of large numbers of close encounters and collisions between planetesimals, as well as (ii) accurate integration for hundreds of millions of years. By exploiting the mathematical structure of the problem, computations can be parallelized to run on CPU/GPU-Clusters.





Chaotic Evolution

Non-linear dynamical systems are inherently chaotic. Small differences in initial conditions rapidly grow, and eventually diverge. But what is the relevance of chaos in simulations of terrestrial planet formation?

Using the N-Body code Genga (Grimm+ 2014), we evolve a suite of initially identical planetesimal disks. They consist of 2048 equal mass planetesimals. On the left, we show initial and final states for three representative runs. Larger circles correspond to more massive objects.

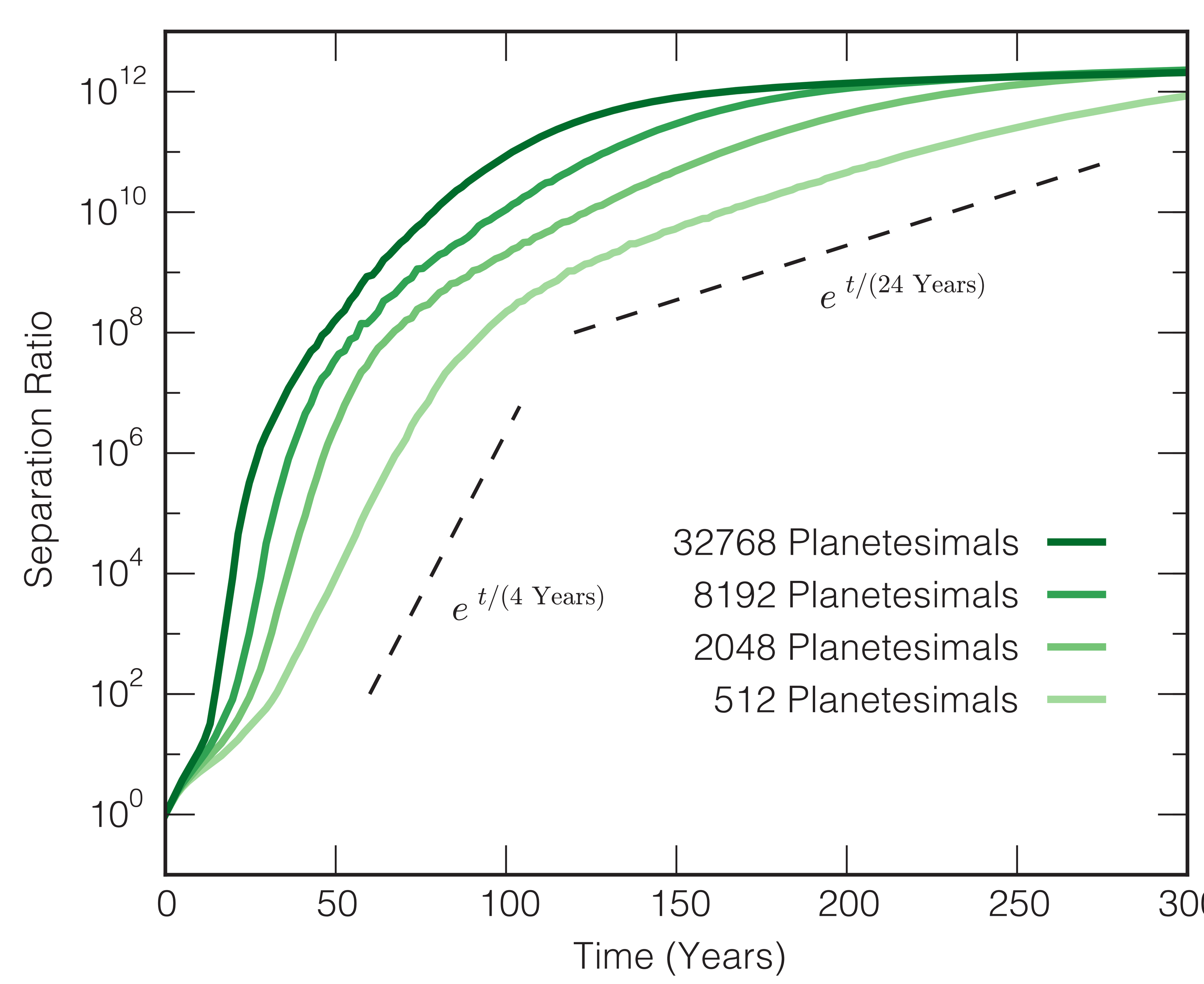
After 148 Myr, individual runs result in a different planetary configurations. Statistical properties (mass/eccentricity distributions) appear consistent up to some spread. We conclude that individual runs lack predictive power, and that a statistical approach is always required.

Exponential Divergence

Computer arithmetic operates at finite precision. If we execute a sequence of operations in different order, the round-off errors differ. In Genga (and other codes), this seeds perturbations leading to chaotic divergence, even if starting from identical initial conditions.

To determine the rate of divergence, we fix the order of operations, and evolve the system from (i) a reference initial condition, and (ii) an initial condition with perturbations at the level of floating point precision. We show the mean orbital separation of planetesimals between the two runs on the right.

Initially nearby orbits diverge exponentially fast with e-folding times on the order of a few years. Adding more planetesimals speeds up divergence. After a few hundred years, separations are on the order of the size of the system, and the separation growth saturates.



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Raymond S. N. et al. 2013, arXiv, 1312.1689v4
Grimm S. L., Stadel J. G., 2014, ApJ, 796, 16